11.3 Arithmetic and Geometric Series

Sequence	Series
1,4,7,10...	1+4+7+10+...
2,6,12,24...	2+6+12+24+...

What is the difference between a sequence and a series?

Series is adding all the terms in a sequence

1) There once was a little boy who was a troublemaker in class. His teacher just wanted a moment of relief, so she gave him the task of adding all of the numbers between 1 and 100. He quickly went to work and she was grateful for the moment of silence from him. Much to her dismay – he quickly announced that he had found the sum: "5,050" he proclaimed! How did this boy do it so quickly?

$1+2+3\ldots+98+99+100$

$50(1+100) = \boxed{5,050}$

Sum of a FINITE arithmetic series (ends)	$\frac{n}{2}(a_1+a_n) = \frac{n(a_1+a_n)}{2}$ (last term)
	n = # of terms

* use explict formula to solve for number of terms (n)

Find the sum of 4 + 9 + 14 + ... + 99.

$a_n = 4+5(n-1)$

$99 = 4+5(n-1)$

$95 = 5(n-1)$

$19 = n-1$

$n = 20$

$S_{20} = \frac{20(4+99)}{2} = \boxed{1030}$

3) The first four rows of chairs are set up for a meeting. The first row has 4 chairs and each consecutive row increases by 2. The seating pattern is to continue through 15 rows. How many chairs will there be in all 15 rows?

$d = 2 \quad a_1 = 4$

$a_n = 4+2(n-1)$

$a_{15} = 4+2(15-1) = 32$

$S_{15} = \frac{15(4+32)}{2} = \boxed{270 \text{ seats}}$

4) A company pays ~~a $10,000 bonus~~ to salespeople at the end of their first 50 weeks if they make 10 sales in their first week, and then improve their sales numbers by 2 each week thereafter. One sales person qualified for the bonus with the minimum possible number of sales.

a) How many sales did the salesperson make in week 50?

b) How many sales did the salesperson make in all 50 weeks?

c) If the salesperson earns a $57 commission per sale, what would their income be at the end of the 50 weeks?

Summation Notation

You can use the Greek capital letter sigma, $\sum$, to indicate a sum.

upper limit (end) term #

$\sum$ explicit formula

lower limit (start) term #

For example, you can write the series $1^2 + 2^2 + 3^2 + \cdots + 108^2$ as $\sum_{n=1}^{108} n^2$

5) What is the sum of the series written in summation notation?

a) $\sum_{n=1}^{70}(5n+3) = \frac{70(8+353)}{2} = 12635$

$a_1 = 5(1)+3 = 8$

$a_{70} = 5(70)+3 = 353$

b) $\sum_{n=1}^{7}(n-1)^2 = 91$

6) Write each series in summation notation.

a) 7 + 11 + 15 + ... + 203 + 207

$\sum_{n=1}^{51} 7+4(n-1)$

$207 = 7+4(n-1)$

$200 = 4(n-1)$

$50 = n-1$

$51 = n$

b) -5 + 2 + 9 + 16 + ... +261 + 268

$\sum_{n=1}^{40} -5+7(n-1)$

$268 = -5+7(n-1)$

$273 = 7(n-1)$

$39 = n-1$

$40 = n$

start at 1st term

Sum of a FINITE geometric series	$S_n = \frac{a_1(1-r^n)}{1-r}$ or graphing calculator (instructions on back of last page)

7) Find the sum of each finite geometric series.

a) 3 + 6 + 12 + 24 + ... + 3072

$a_1 = 3 \quad r = 2$

$\sum_{n=1}^{11} 3 \cdot 2^{n-1} = 6141$

$3072 = 3 \cdot 2^{n-1}$

$1024 = 2^{n-1}$

$2^{10} = 2^{n-1}$

$10 = n-1$

$11 = n$

b) $\sum_{n=0}^{20} 4\left(\frac{1}{2}\right)^n \approx 8$

c) $\sum_{n=0}^{10} 5(-2)^{(n-1)}$

d) $-15 + 30 - 60 + 120 - 240 + 480$

8) A famous story involves a soldier who rescues his king in battle. The king grants him any prize "within reason" from the riches of the kingdom. The soldier asks for a chessboard with a single kernel of wheat on the first square, two on the second square, then four, then eight, and so on for all 64 squares of the chessboard. The king decides that the request is reasonable. How many kernels of wheat did the soldier request?

$a_1 = 1 \quad r = 2 \quad n = 64$

$\sum_{n=1}^{64} 1 \cdot (2)^{n-1}$

1.84×10^{19} kernals

9) To save money for your senior trip, you set aside \$100. For each month thereafter, you plan to set aside 10% more than the previous month. How much money will you save in 12 months?

The terms of a geometric series grow rapidly when the common ratio is greater than 1. Likewise, they diminish rapidly when the common ratio is between 0 and 1. In fact, they diminish so rapidly that an **INFINITE GEOMETRIC SERIES** has a finite sum.

Sum of an INFINITE geometric series *	$S_\infty = \frac{a_1}{1-r}$
↑ doesn't end ...	↓ Converges: $\lvert r\rvert < 1$ Diverges: $\lvert r\rvert > 1$

10) Determine if the series converges or diverges. If it converges, state the sum ~~say to which number~~.

a) $1 + .5 + .25 + \cdots$

$r = \frac{1}{2}$ $a_1 = 1$ Converges to 0

$S_\infty = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

b) $\sum_{n=0}^{\infty} 5\left(-\frac{4}{3}\right)^n$ Diverges

$r = \left|\frac{-4}{3}\right| > 1$

c) $.125, -.25, .5, -1, \ldots$

Diverges

$r = -2$

d) $9 + 3 + 1 + \cdots$

Converges $a_1 = 9 \quad r = \frac{1}{3}$

$S_\infty = \frac{9}{1-\frac{1}{3}} = \frac{9}{\frac{2}{3}} = \frac{27}{2} = 13.5$

e) $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$

f) $\sum_{n=0}^{\infty} -\left(\frac{2}{3}\right)^n$

c, w

Using a graphing calculator to find the (sum of a sequence) series

[2nd] [stat] → Math; 5: sum

[2nd] [stat] → Ops, 5: seq

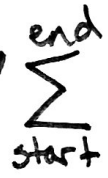
sum(seq(expression, variable, start, end, step))

Expression: Explicit formula (use x instead of n)

Variable: X

Start:

End:



Step: 1