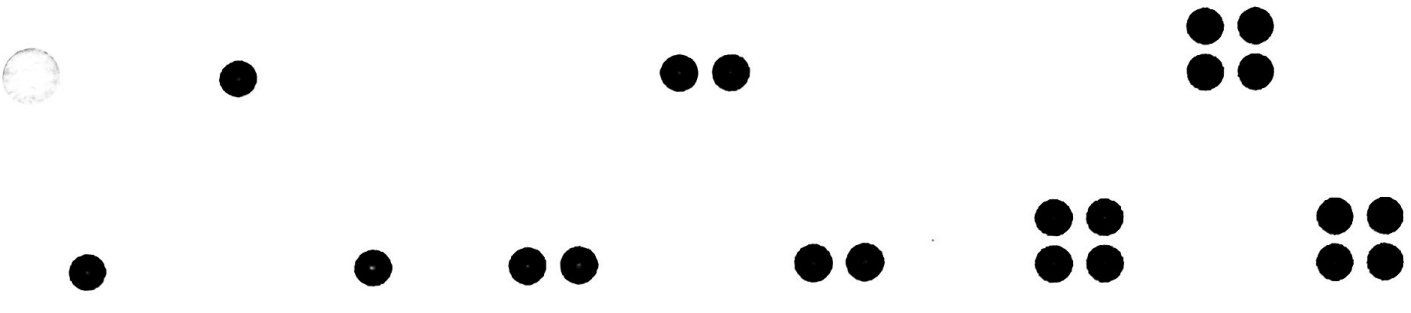
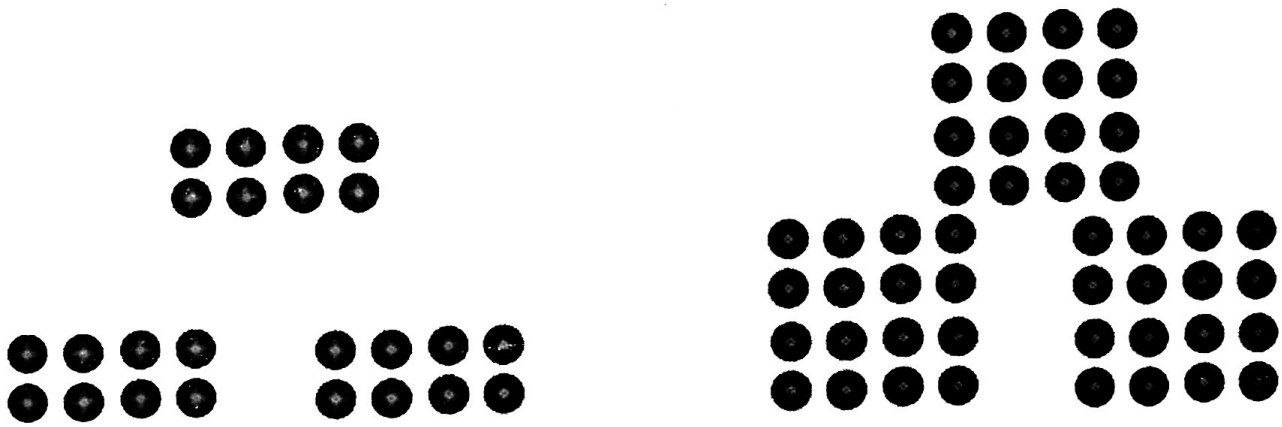
11.2 Geometric Sequences



Phase 1 Phase 2 Phase 3



Phase 4 Phase 5

1) Describe the pattern that you see above.

Doubles each time

2) Write a recursive formula that describes the pattern above. Write an explicit formula that describes the pattern above.

$a_n = 2a_{n-1}$ $a_n = 3(2)^{n-1}$

3) How many dots will there be at phase 7? Phase 15?

$a_7 = 3(2)^{7-1} = 3 \cdot 2^6 = \boxed{192}$ $a_{15} = 3(2)^{15-1} = \boxed{49152}$

Geometric sequence: Sequence that multiplies by something each time

Common ratio: What you multiply by each time

4) Determine if the sequence is geometric or not. If so, find the next three terms in the sequence.

a) 3, 6, 12, 24, 48, ...

$r = 2$ 96, 192, 384

b) 3, 6, 9, 12, 15, ...

Not geometric

c) $3^5, 3^8, 3^{11}, 3^{14}, \ldots$

$r = 3^3$ $3^{17}, 3^{20}, 3^{23}$

d) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots$

$r = \frac{-1}{2}$ $\frac{-1}{32}, \frac{1}{64}, \frac{-1}{128}$

Top: -1
Bottom: 2

With fractions, you can look at top & bottom separately to find common ratio

GEOMETRIC SEQUENCES	
Recursive Definition	**Explicit Formula**
$a_n = r\,a_{n-1}$ r = common ratio *need to define a_1	$a_n = a_1 \cdot r^{n-1}$ $r = \frac{a_2}{a_1}$

5) Write the explicit and recursive formula for each geometric sequence.

a) 4, 12, 36...

$r = 3$

$a_1 = 4$

R: $a_n = 3a_{n-1}$, $a_1 = 4$

E: $a_n = 4 \cdot 3^{n-1}$

b) -2,6,-18,54...

$r = -3$

$a_1 = -2$

R: $a_n = -3a_{n-1}$, $a_1 = -2$

E: $a_n = -2(-3)^{n-1}$

c) $3, 2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}$...

Top: ×2

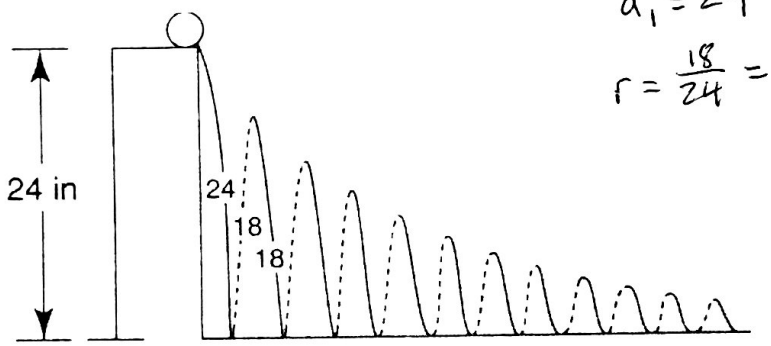
Bottom: ×3

$r = \frac{2}{3}$

R: $a_n = \frac{2}{3}a_{n-1}$, $a_1 = 3$

E: $a_n = 3\left(\frac{2}{3}\right)^{n-1}$

6) When a ball bounces, the heights of the consecutive bounces form a geometric sequence. What are the heights of the 8th and 9th bounces?



$a_1 = 24$

$r = \frac{18}{24} = \frac{3}{4}$

$a_n = 24\left(\frac{3}{4}\right)^{n-1}$

$a_8 = 24\left(\frac{3}{4}\right)^{8-1} = 3.2$ in

$a_9 = 24\left(\frac{3}{4}\right)^{9-1} = 2.4$ in

7) Find the missing terms for each geometric sequence.

a) ...8, 24, 72, 216,...

×3 ×3

$\frac{72}{8} = 9$ $\sqrt{9} = 3$

b) 3, 15, 75, 375,...

×5 ×5 ×5

$\frac{375}{3} = 125$ $\sqrt[3]{125} = 5$

c) -2, -12, -72, -432, -2592,...

×6 ×6 ×6 ×6

$\frac{-2592}{-2} = 1296$ $\sqrt[4]{1296} = 1296^{\frac{1}{4}} = 6$

d) $-1, \frac{4}{5}, -\frac{16}{25}, \frac{64}{125}$,...

Top: ×−4

Bottom: ×5

$r = -\frac{4}{5}$

8) The 9th and 11th term of a geometric sequence are 45 and 80. What are the possible values for the 10th term?