1.5 Properties of Rational Exponents

Sometimes we see an exponent in the form of a fraction. This means that we can rewrite it into a **radical expression**. The denominator of the fraction signifies what kind of root you are taking. The numerator signifies the exponent of the base inside the root.

Ex: $3^{\frac{1}{2}} = \sqrt{3}$ $\qquad$ $m^{\frac{1}{5}} = \sqrt[5]{m}$ $\qquad$ $y^{\frac{2}{3}} = \sqrt[3]{y^2}$ $\qquad$ $2x^{\frac{2}{5}} = 2\sqrt[5]{x^2}$

2) Convert each fractional exponent to a radical expression or vice versa.

a. $x^{\frac{4}{5}}$ $\quad \sqrt[5]{x^4}$

b. $3a^{\frac{1}{3}}$ $\quad 3\sqrt[3]{a^1}$

c. $(3a)^{\frac{2}{3}}$ $\quad \sqrt[3]{(3a)^2} = \sqrt[3]{3^2a^2} = \boxed{\sqrt[3]{9a^2}}$

d. $\sqrt{5} = \sqrt[2]{5^1} = 5^{\frac{1}{2}}$

e. $\sqrt[4]{2x^5} = \sqrt[4]{2^1 x^5} = \boxed{2^{\frac{1}{4}}x^{\frac{5}{4}}}$

f. $\sqrt[3]{(6x)^4}$ $\quad (6x)^{\frac{4}{3}}$

$*$ $\sqrt{}$ really means $\sqrt[2]{}$

Now that you know what a fractional exponent means, we're going to practice simplifying with fractional exponents.

Same rules as 1.4, just with fractions

3) Simplify. Your answer should contain only positive exponents.

a. $(m^{\frac{1}{2}})^{\frac{3}{1}}$

$\boxed{m^{\frac{3}{2}}}$

b. $(m^{\frac{1}{2}} \cdot m^{\frac{2}{5}})^4$

$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

$\left(m^{\frac{9}{10}}\right)^{\frac{4}{1}}$

$m^{\frac{36}{10}} = \boxed{m^{\frac{18}{5}}}$

Reduce fractions when you can

c. $\dfrac{x^{\frac{3}{5}}}{x^{\frac{1}{2}}}$

$\frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

$\boxed{x^{\frac{1}{10}}}$

d. $\dfrac{3x^{\frac{1}{4}}}{x^{\frac{1}{2}}}$

$\frac{1}{4} - \frac{1}{2} = \frac{1}{4} - \frac{2}{4} = -\frac{1}{4}$

$\dfrac{3x^{-\frac{1}{4}}}{1} = \boxed{\dfrac{3}{x^{\frac{1}{4}}}}$

e. $(2y^{\frac{1}{2}} \cdot 6y^{\frac{2}{5}})^4$

$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

$\left(12y^{\frac{9}{10}}\right)^{\frac{4}{1}}$

$12^4 y^{\frac{36}{10}} = \boxed{12^4 y^{\frac{18}{5}}}$

f. $\dfrac{(4x^{\frac{1}{4}})^2}{x^{\frac{1}{2}}}$

$\dfrac{4^2 x^{\frac{2}{4}}}{x^{\frac{1}{2}}} = \dfrac{16x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = \boxed{16}$

Fraction Rules

Add/Sub: Common denominator
ex: $\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

Multiply: Multiply across
ex: $\frac{2}{5} \cdot \frac{3}{7} = \frac{6}{35}$