1.4 Properties of Integer Exponents

An **exponent** a number that tells us how many times a quantity is multiplied by itself. Another word for exponent is **power**. The quantity that is being multiplied by itself is called the **base**.

Ex: $5^3 = 5 \cdot 5 \cdot 5$ $\quad 7^2 = 7 \cdot 7$ $\quad 21^4 = 21 \cdot 21 \cdot 21 \cdot 21$

Using this information, see if you can figure out some shortcuts or rules for simplifying exponents. Be sure to show your work to help you.

Product of Powers

Simplify the following exponents:

$6^3 \cdot 6^5 = 6^8$ $\quad (6 \cdot 6 \cdot 6)(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)$

$4^7 \cdot 4^2 = 4^9$ $\quad 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \quad 4 \cdot 4$

$12^4 \cdot 12^4 = 12^8$ $\quad (12 \cdot 12 \cdot 12 \cdot 12)(12 \cdot 12 \cdot 12 \cdot 12)$

Quotient of Powers

Simplify the following exponents:

$\frac{2^7}{2^4} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = \boxed{2^3}$

$\frac{8^5}{8^2} = \frac{\cancel{8} \cdot \cancel{8} \cdot 8 \cdot 8 \cdot 8}{\cancel{8} \cdot \cancel{8}} = \boxed{8^3}$

$\frac{10^9}{10^4} = \frac{\cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{\cancel{10} \cdot \cancel{10} \cdot \cancel{10} \cdot \cancel{10}} = \boxed{10^5}$

Power of Powers

Simplify the following exponents:

$(2^3)^3 = (2^3 \cdot 2^3 \cdot 2^3) = \boxed{2^9}$

$(5^2)^4 = \boxed{5^8}$ $\quad 5^2 \cdot 5^2 \cdot 5^2 \cdot 5^2$

$(8^4)^3 = \boxed{8^{12}}$ $\quad 8^4 \cdot 8^4 \cdot 8^4$

Property	Notation	Rule	Examples
Product of Powers	$x^m \bullet x^n$	Keep same base, add exponents	$a^4 \cdot a^3 = a^7$ $5x^2 \cdot 2x^9 = 10x^{11}$ *multiply coefficients
Quotient of Powers	$\frac{x^m}{x^n}$	Keep same base, subtract exponents	$\frac{a^7}{a^2} = a^5$ $\frac{6x^{10}}{2x^{-1}} = 3x^{11}$ $10-(-1)$ $10+1$
Power of Powers	$(x^m)^n$	Keep same base, multiply exponents	$(a^3)^2 = a^6$ $(2x^2)^5 = 2^5 x^{10} = 32x^{10}$

*the exponent applies to everything in the parentheses

Notice that to simplify the exponents, the two expressions **must** have the same base. If you have coefficients in front of your exponent-base pair, multiply them as normal.

There are two other exponent properties that we need to talk about:

Property	Rule	Example	
Zero Property	Anything to the 0 power is 1	$a^0 = 1$	$12^0 = 1$
Negative Exponent Property	Represents a fraction Anything with a negative exponent switches spots in a fraction. Then the exponent becomes positive.	$\frac{x^{-5}}{1} = \frac{1}{x^5}$ $\frac{2a^{-2}}{1} = \frac{2}{a^2}$ $\frac{5}{3m^{-6}} = \frac{5m^6}{3}$	Anything can be made a fraction by putting it over 1.

Now we're going to put everything together. **Make sure to remember the order of operations!**

PEMDAS →

Anything without an exponent has an exponent of 1

1) Simplify. Your answer should only contain positive exponents.

a. $(2b^4)^0$

$\boxed{1}$

0 exponent is attached to everything, so everything becomes 1

b. $(2b)^4$

$2^4 b^4$

$\boxed{16b^4}$

c. $(2m^2 \cdot 3m)^2$

$(6m^3)^2$

$6^2 m^6$

$\boxed{36m^6}$

d. $\frac{(b^4)^2}{b^5}$

$= \frac{b^8}{b^5} = \boxed{b^3}$

e. $\frac{5a^7}{4a^4}$

$\boxed{\frac{5a^3}{4}}$

f. $\frac{(3x^2)^{-3}}{1}$

$\frac{1}{(3x^2)^3} = \frac{1}{3^3 x^6}$

$= \boxed{\frac{1}{27x^6}}$

g. $\frac{6y^4 \cdot 3y^2}{2y^3}$

$\frac{18y^6}{2y^3} = \boxed{9y^3}$

h. $\frac{6x^{-2}}{2x^4}$

$\frac{6}{2x^4x^2} = \frac{6}{2x^6}$

$= \boxed{\frac{3}{x^6}}$

i. $\frac{(3m^3)^2}{2m^{-4}}$

$= \frac{3^2 m^6}{2m^{-4}} = \frac{9m^6}{2m^{-4}}$

$= \frac{9m^6m^4}{2} = \boxed{\frac{9m^{10}}{2}}$